Analysis of the Kobe earthquake time series via system identification and fault-detection techniques

S. Bittanti y S. Garatti

Dipartimento di Elettronica, Informazione e Bioingegneria
Politecnico di Milano, Piazza L. da Vinci, 32, 20133 Milano, Italy.
sergio.bittanti@polimi.it
simone.garatti@polimi.it

ABSTRACT

The Kobe earthquake was one of the most severe earthquakes in Japan in recent years. It occurred on January 16, 1995, at 20:46:49 (UTC) and measured 6.8 on the moment magnitude scale. In this paper, the time series of the unbiased earth ground vertical acceleration collected by a seismograph located at the University of Tasmania, Hobart, Australia, is analyzed. The time series is segmented into three consequent sub-series which represent the normal seismic activity before the arrival of the earthquake, a transition phase, and the arrival of earthquake waves. The analysis is separately performed for each segment. We show that by inspecting the degradation of the prediction performance of the model identified based on the normal seismic activity data set, it is possible to distinguish between the transition phase and normal seismic activity about 200-300 seconds before the beginning of the earthquake phase. Though this does not mean that earthquakes can be forecasted, because of the significant data distortion due to the long distance between the epicenter and the data collection location, nevertheless the achieved result may open up new routes in the study of earthquakes.

Key words: seismology, earthquakes, time series prediction, model identification, fault detection.

Introduction

The Kobe earthquake occurred on January 16, 1995, at 20:46:49 (UTC) and measured 6.8 on the moment magnitude scale. It was one of the most severe earthquakes in Japan in recent years. In this paper, we analyze the time series of the unbiased vertical acceleration in nm/s² of the earth ground collected by a seismograph located at the University of Tasmania, Hobart, Australia, over a time window including the arrival of seismic waves generated by the Kobe earthquake. To be precise, the meas-
measurements began at 20:56:51 (UTC), with a sampling time $T=1$ s, and lasted for 3000 seconds (about 51 minutes). In the following, the time series will be denoted by $y(t)$, $t=1, 2, ..., 3000$. See Figure 1 for a plot of $y(t)$.

As is clear from the sudden change of the time series amplitude, the seismic waves show up 1700 seconds after the beginning of data recording, that is, at time 21:25:10 (UTC). The 40 minute delay with respect to the earthquake occurrence is clearly due to the 8600 km distance between Kobe and Hobart. It results in a propagation speed of about 3580 m/s, which is quite likely for seismic waves.

Because of the arrival of the seismic waves, visual inspection reveals that the time series cannot be thought of as a realization of a stationary stochastic process (variance at least is time-varying). For this reason, we decided to segment the time series into three parts as shown in Figure 2.

The first segment for $t=1, ..., 1200$ is labelled as “normal seismic activity”, since seismic waves have not arrived yet, whilst the third segment for $t=1601, ..., 3000$ is labelled as “earthquake” for the very opposite reason. The second segment for $t=1201, ..., 1600$, instead, represents a “transition phase” between the normal seismic activity and the earthquake phase. Recalling that the time series is unbiased and hence the mean of $y(t)$ is equal to 0, the empirical variance calculated over the second segment is

$$\frac{1}{400} \sum_{t=1201}^{1600} y(t)^2 = 2.5 \times 10^7 \text{ (nm/s}^2)^2$$

Importantly enough, this variance has the same magnitude as that registered during the normal seismic activity, which is

$$\frac{1}{1200} \sum_{t-1}^{1600} y(t)^2 = 1.15 \times 10^7 \text{ (nm/s}^2)^2$$

![Figure 1. The time series $y(t)$.](image)

**Figure 1.** The time series $y(t)$.

**Figura 1.** La serie temporal $y(t)$. 
Hence, during the transition phase no oscillations with a magnitude different from that of oscillations in the normal seismic activity phase are perceived. During the earthquake segment, instead, the variance is

\[ \frac{1}{1400} \sum_{t=1001}^{3000} y(t)^2 = 1.12 \times 10^8 \ (\text{nm}^2/\text{s}^2) \]

which is one order of magnitude bigger than that of the previous segments.

The objective of this paper is to show that, though the empirical variance is the same, the transition phase and the normal seismic activity are radically different in terms of the underlying generation mechanism. In fact, after performing model identification over the normal seismic activity segment, we will show that, by observing the degradation of the model prediction capabilities, it is possible to discern between the normal seismic activity and the transition phase in spite of the lack of variation of the amplitude of oscillations. This opens up the possibility of forecasting the subsequent earthquake phase in the collected data.

The paper is organized as follows. Model identification, including model order selection, of the normal seismic activity segment is discussed in the section Modelling the normal seismic activity segment, while the Model validation section provides some model validation over both the normal seismic activity segment and the earthquake segment. The Analysis of the transition phase via fault detection techniques section applies fault-detection techniques to the transition phase segment and shows that the degradation of the prediction capabilities of the model identified in the Modelling the normal seismic activity segment

![Figure 2. Partition of the time series into three segments.](image)

**Figure 2.** Partition of the time series into three segments.

**Figura 2.** Partición de la serie temporal en tres segmentos.
section permits us to discern the transition phase from the normal seismic activity 200-300 seconds before the beginning of the earthquake phase. Finally, some conclusions are drawn in the Conclusions section.

Modelling the normal seismic activity segment

Non-parametric statistical analysis was first performed, see (Anderson, 1959), (Ljung, 1989), (Brockwell and Davis, 2002), (Stoica and Moses, 2005), and (Bittanti, 2017). Figure 3 displays the periodogram

\[ \hat{\gamma}_y(\omega) = \frac{1}{1200} \sum_{t=1}^{1200} y(t) e^{-j \omega t} \]

along with the empirical covariance function

\[ \hat{\gamma}_y(\tau) = \frac{1}{1200 - \tau} \sum_{\tau=1}^{1200-\tau} y(i) y(i+\tau) \]

and the partial correlation function \( \text{parcor}(k) \) for the time series \( y(t) \) over the normal seismic activity segment. \( \text{parcor}(k) \) was calculated by means of the Durbin-Levinson algorithm, see (Durbin, 1959).

Being \( \hat{\gamma}_y(\tau) \) and \( \text{parcor}(k) \) not null after a finite number of time lags, not even approximately, Figure 3 suggests modelling the time series in the normal seismic activity segment as the realization of an ARMA \((n_a, n_c)\) stochastic linear model:

\[
y(t) = \frac{C(z)}{A(z)} e(t) = \frac{1 + c_1 z^{-1} + \cdots + c_{n_a} z^{-n_a}}{1 + a_1 z^{-1} + \cdots + a_{n_c} z^{-n_c}} e(t), \quad e(t) = \text{WN}(0, \hat{\sigma}^2)
\]

\((\text{WN} = \text{white noise}).\)

For the fixed model orders \( n_a, n_c \) the model parameters are retrieved by means of standard Prediction Error (PE) methods, (Ljung, 1989), (Sodestrom and Stoica, 1989), (Box et al, 2016), and (Bittanti, 2017). That is, letting \( \theta = [a_1 \ldots a_{n_c} c_1 \ldots c_{n_a}]' \), the optimal parameter vector is obtained as the minimizer of the empirical prediction error variance:

Figure 3. Non-parametric properties of \( \gamma(t) \) over the normal seismic activity segment.

Figura 3. Propiedades no-paramétricas de \( \gamma(t) \) sobre el segmento de actividad sísmica normal.
where
\[ y(t | t - 1, \theta) = \frac{C(z)}{C(z)} y(t) \]
is the 1-step optimal predictor for model in equation (1). The value
\[ \hat{\theta}_{1200}^{n_a, n_c} \]
can be actually computed based on classical quasi-Newton algorithms, (Ljung, 1989), (Söderström and Stoica, 1989), and (Bittanti, 2017).

As for the optimal model orders, we resort to Rissanen’s Minimum Description Length (MDL) indicator, (Rissanen, 1978), (Grunwald, 2007), and (Bittanti, 2017). To be precise, we let both \( n_a \) and \( n_c \) range from 1 up to 12, and compute
\[ \text{MDL}(n_a, n_c) = \ln(1200) \frac{n_a + n_c}{1200} + \ln \left( \frac{1}{1200} \sum_{t=1}^{1200} (y(t) - \hat{y}(t | t - 1, \hat{\theta}_{1200}^{n_a, n_c}))^2 \right) \]

The optimal \( n_a, n_c \) are those returning the lowest value for MDL.

The actually computed values for MDL are reported in Figure 4, showing that the minimum is achieved for \( n_a = 7 \) and \( n_c = 9 \).

The identified model, corresponding to
\[ A(z) = 1 - 2.5 z^{-1} + 2.8 z^{-2} - 1.6 z^{-3} + 0.7 z^{-4} - 0.6 z^{-5} + 0.2 z^{-6} \]
\[ C(z) = 1 + 1.1 z^{-1} - 1.6 z^{-2} - 1.7 z^{-3} + 1.6 z^{-4} + 1.2 z^{-5} - 1.5 z^{-6} - 1.0 z^{-7} + 0.6 z^{-8} + 0.4 z^{-9} \]
is given by
\[ \hat{\theta}_{1200}^{7,9} \]
for the various \( n_a \) and \( n_c \) according to equation (2). For each identified model parameter vector, the MDL indicator is computed as
\[ \text{MDL}(n_a, n_c) = \ln(N) \frac{n_a + n_c}{N} + \ln(J(\hat{\theta}_{1200}^{n_a, n_c})) \]

The identified model, corresponding to
\[ y(t) = \hat{\theta}_{1200}^{7,9} \]
is given by
\[ n_a | n_c \]
1 14,987 14,153 13,635 13,538 13,252 13,365 13,547 13,860 13,631 14,110 14,357 14,018
2 14,010 13,317 13,087 13,262 13,306 13,462 13,640 13,840 13,929 14,150 14,357 14,541
3 13,563 13,162 13,082 13,050 13,308 13,378 13,658 13,759 13,987 14,204 14,383 14,458
4 13,378 13,145 13,105 13,100 13,133 13,405 13,538 13,791 13,982 14,212 14,389 14,241
5 13,214 13,193 13,236 13,249 13,192 13,202 13,568 13,841 14,016 14,226 14,413 14,432
6 13,104 13,394 13,210 13,359 13,315 13,475 13,682 13,847 14,041 14,222 14,079 14,153
7 13,212 13,576 13,417 13,516 13,481 13,712 13,576 13,807 12,124 14,166 14,230 14,070
8 13,370 13,695 13,671 13,821 13,775 13,735 13,781 12,130 12,137 12,141 14,086 12,160
9 12,301 13,640 12,132 12,136 12,141 12,129 12,120 12,139 12,142 12,152 12,144 12,160
10 12,253 12,245 12,144 12,148 12,152 12,118 12,143 12,137 12,134 12,136 12,156 12,160
11 12,236 12,198 12,151 12,139 12,150 12,137 12,142 12,144 12,136 12,149 12,154 12,162
12 12,232 12,180 12,159 12,159 12,155 12,160 12,165 12,169 12,174 12,147 12,154 12,160

Figure 4. MDL(n_a, n_c) for n_a = 1, ..., 12 and n_c = 1, ..., 12.
Figura 4. MDL(n_a, n_c) for n_a = 1, ..., 12 y n_c = 1, ..., 12.
Figure 5 depicts the spectrum
\[
\Gamma_y(\omega) = \frac{|C(e^{-j\omega\hat{\theta}_{1200}})|^2}{|A(e^{-j\omega\hat{\theta}_{1200}})|^2} \lambda^2
\]

of the ARMA stochastic process generated by (3).
This spectrum is in full agreement with the periodogram in Figure 3. The poles (crosses) and zeros (circles) of (3) are also reported in Figure 5.

Model validation
As recalled by the saying “the proof of the pudding is in the eating”, model validation is performed to test the model capabilities. Based on the model corresponding to (3), the predictor
\[
\hat{y}(t \mid t-1, \hat{\theta}_{1200}^{7.9})
\]
can be computed. The left side of Figure 6 depicts \( y(t) \) vs.
\[
\hat{y}(t \mid t-1, \hat{\theta}_{1200}^{7.9})
\]
over the normal seismic activity segment, i.e. for \( t=1, \ldots, 1200 \). The corresponding prediction error
\[
\varepsilon(t \mid t-1, \hat{\theta}_{1200}^{7.9}) = y(t) - \hat{y}(t \mid t-1, \hat{\theta}_{1200}^{7.9})
\]
has empirical variance equal to
\[
\frac{1}{1200} \sum_{t=1}^{1200} \varepsilon(t \mid t-1, \hat{\theta}_{1200}^{7.9})^2 = \lambda^2 = 1.59 \cdot 10^5 \text{ (nm/s)^2}
\]
which is two order of magnitude smaller than the empirical variance of \( y(t) \).
The right side of Figure 6 depicts the correlation coefficient
\[
\hat{\rho}_r(\tau) = \frac{\hat{y}_r(\tau)}{\hat{y}_r(0)}
\]
along with the 95 % confidence interval for the Anderson’s whiteness test, (Anderson, 1959), (Ljung, 1989), and (Box et al., 2016). Since all the displayed \( \hat{\rho}_r(\tau), \tau=1, \ldots, 24, \) are within the confidence interval, the whiteness test is passed and
\[
e(t \mid t-1, \hat{\theta}_{1200}^{7.9})
\]
can be presumed to be a white noise. Thus, the conclusion is drawn that the identified model is an accurate and complete descriptor of the data generation mechanism underlying the registered normal seismic activity.

We then checked whether the obtained model is also a good descriptor of the earthquake segment, for \( t=1601, \ldots, 3000, \) or not.
A negative answer is quite immediate because even both the plot of \( y(t), t=1601, \ldots, 3000, \) (Figure 7, left) and that of the corresponding periodogram (Fig.7, right) present features that are remarkably different from those of the same plots during normal
seismic activity. Anyway, by applying the whiteness test to the prediction error

\[ v(t | t-1, \hat{\theta}_{1200}^{7.9}) \]

for \( t=1601, \ldots, 3000 \), the result shown in Figure 8 is obtained.

This clearly reveals that the model corresponding to (3) is not apt for describing the earthquake phase and that earthquakes differ from the normal seismic activity not only because of the amplitude of oscillations but also because a change of the underlying data generation mechanism.
The idea is borrowed from the field of fault detection/fault diagnosis, (Hwang et al, 2010), and is easily explained as follows.

We have already seen that model (3), which, we may recall, has been identified during the normal seismic activity phase, is able to discern between the normal seismic activity and the earthquake phase, because in the first case the returned prediction error is white (the whiteness test is passed) whilst in the second case it is not (the whiteness test is failed). As is clear, this property is not very useful because the two segments are discerned by means of the variation of the amplitude of oscillations (as pointed out in the introduction, there is a variation of one order of magnitude between the empirical variance of the time series in the first and in the third segment). The transition phase, instead, presents oscillations whose amplitude is close to that of oscillations in the normal seismic activity. The question is whether the use of model (3) permits us to discern the first and the second segments or not.

To answer this question, we further divided the transition phase segment into the four 100 seconds long time windows as shown in Figure 9, and performed the whiteness test to the prediction error sequence

\[ \varepsilon(t) \mid t-1, \hat{\theta}_{100}^* \]
achieved for each time window. The results are shown in Figure 10.

As is clear, the whiteness test is passed in just the first time window (though the values of $\rho_e(\tau)$ present a suspicious regularity). The whiteness test is not passed in the other time windows, and this becomes clearer and clearer as the windows get closer to the earthquake phase.

Conclusions

The results of the Analysis of the transition phase via fault detection techniques section show that, in the collected data, it is possible to discern the transition phase from the normal seismic activity phase about 200-300 seconds before the beginning of the earthquake phase. This is achieved by looking at the degradation of the prediction capabilities of the model identified during the normal seismic activity phase over the transition phase segment. In particular, a whiteness test for the prediction error has been used to this purpose.

Clearly, the main limitation of this analysis is that it is carried out on data collected in a location far away from the earthquake epicenter, so that earthquake waves are registered after distortion due to the propagation through the earth. This means that the phenomenon studied by means of the available data is quite different from that perceived in the proximity of the earthquake’s epicentre, and thus the analysis here reported in no way allows us to say that earthquakes can be forecasted.

Nonetheless, we still believe that the achieved
result is interesting and that it may open up some further developments in the study of earthquakes.

**Acknowledgements**

Paper supported by Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR) and by CNR-IEIIT.

**References**


Recibido: septiembre 2016
Revisado: diciembre 2016
Aceptado: enero 2017
Publicado: septiembre 2018